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MULTILEVEL OPTIMIZATION OF ARRAYS OF PROTECTIVE STRUCTURES

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Summary

The present study deals with optimal design of arrays of protective structures. The structures, intended for storage of explosive materials, consist of rectangular reinforced concrete (RC) plates, beams, and doors.

The design variables include: a) The elements' cross sectional dimensions; b) The structural configuration; c) The glometric location of the structure. The constrains are related to safety distances, functional requirements, and structural behavior. The objective function represents the cost, including cost of materials, real estate, subgrade, maintenance, etc.

The main difficulties involved in this problem stem from the complex analysis and the nature of the design variables, constraints, and objective function. The various types of variables are of fundamentally different nature, and some of the variables are discrete. The objective function is neither differentiable nor continuous.

Approximate behavior models are employed in order to simplify the analysis. Since it is not practical to optimize all the design variables simultaneously, it is proposed to use a multilevel optimization procedure. The variables are optimized in different levels, according to their type and nature. Considerations for choosing the levels are discussed and numerical examples illustrate the approach and its practicality.

1. Introduction

Protective structures intended to resist the effects of explosions, are usually expensive due to the nature of the loadings and design criteria. Therefore, improved designs may lead to considerable savings in the total cost of the structure. While much work has been done in the last two decades on optimum structural design [1,2], most applications are limited to cross sections optimization. Examples of previous work on optimal design of protective structures include: cross sections design [3] or optimization of geometry and cross sections [4] of RC slabs subjected to impulse loadings; and optimal (average) strength of magazine doors [5].

The main difficulties involved in design of protective structures stem from the complex analysis and the nature of the design variables, constraints, and objective function. In general, the nonlinear dynamic analysis must be repeated many times and the various types of variables involved are often of fundamentally different nature, from both the mathematical and the physical points of view.

The present study deals with optimal design of arrays of protective structures, intended for storage of explosive materials. The structure consists of RC rectangular plates, and the design variables include:
(a) The elements cross sectional dimensions (plate thicknesses and amounts of steel);
(b) The structural configuration (length and width);
(c) The geometric location of the structure (distances between adjacent structures).

Due to functional requirements the configurational variables are discrete and the objective function, representing the cost, is neither differentiable nor continuous.

To simplify the analysis, approximate behavior models are employed. These include empirical loading expressions, idealized constitutive laws, and simplified dynamic models. A multilevel optimization procedure, in which the different types of variables are treated separately, is proposed. Several authors, e.g. [6], proposed separate design spaces for geometry and cross sections in truss optimal design. In general, such a solution may be viewed as a multilevel approach [2,7,8]. In the formulation presented in this study, the cross sectional dimensions are optimized in the first level for a given configuration and location of the structure. As a result, the structural elements can be optimized independently in a simple manner. Data banks of optimal elements are introduced, to be used in the higher levels of the optimization. In the second level the structural configuration is optimized for a given location of the structure. For each candidate geometry the optimal cross sectional dimensions are chosen from the data banks. The geometric location of the structure is selected in the third level. For each selected location, both the structural configuration and the cross-sections are optimized.

The proposed approach combines efficient suboptimization for cross sectional variables, reduction in the number of design variables optimized simultaneously, and improved convergence.

2. Problem Statement

Consider the array of RC rectangular magazines shown in Fig. 1. The object is to minimize the cost of storing a unit weight of explosive material. It is assumed that the number of structures is large and all the structures are identical; i.e., the optimal design represents a standard magazine. The preassigned parameters include: materials properties, unit costs, height of the structure, and spacings between beams. Uniform cross sectional dimensions of the RC elements and standard steel doors have been assumed. The optimal design problem can be stated as follows: find the vector of design variables $\{X\} = \{D\},\{L\},\{TC\},\{AS\}$ where

$$\{D\} = \{D_1, D_2\} \quad \text{(distance between adjacent structures)}$$

$$\{L\} = \{L_1, L_2\} \quad \text{(length and width of the structure, } L_1 \in \Lambda_1,\ L_2 \in \Lambda_2) \tag{1}$$

$$\{TC\} = \{TC_1, \ldots TC_I\} \quad \text{(thickness of concrete plates)}$$

$$\{AS\} = \{AS_1, \ldots AS_I\} \quad \text{(amount of steel in plates)}$$

such that

$$C = \sum_{j=1}^{J} C_j / W \rightarrow \min. \quad \text{(objective function)} \tag{2}$$

$$\{D^L\} \leq \{D\} \leq \{D^U\} \quad \text{(bounds on distances)} \tag{3}$$

$$\{L^L\} \leq \{L\} \leq \{L^U\} \quad \text{(bounds on structural geometry)} \quad (4)$$

$$\{TC^L\} \leq \{TC\} \leq \{TC^U\} \quad \text{(bounds on concrete thickness)} \quad (5)$$

$$\{\rho^L\} \leq \{\rho\} \leq \{\rho^U\} \quad \text{(bounds on steel percentage)} \quad (6)$$

$$\{V\} \leq \{V^U\} \quad \text{(deflection constraints)} \quad (7)$$

$$\{\theta\} \leq \{\theta^U\} \quad \text{(rotation constraints)} \quad (8)$$

$$\{\sigma^L\} \leq \{\sigma\} \leq \{\sigma^U\} \quad \text{(bending stress constraints)} \quad (9)$$

$$\{\tau^L\} \leq \{\tau\} \leq \{\tau^U\} \quad \text{(shear stress constraints)} \quad (10)$$

In the above formulation

Λ_1, Λ_2 = sets of allowable discrete values of L_1 and L_2, respectively
I = number of elements in the structure
L, U = superscripts denoting lower and upper bounds, respectively
C = total cost of a magazine per unit stored explosive material
W = quantity of explosive material in a magazine
C_j = the j-th component of the objective function
J = number of components of the objective function
ρ = steel percentage
V = deflection
θ = rotation at the supports
σ = bending stress
τ = shear stress

It should be noted that $\{D^L\}$ is a function of W. The behavior constraints (Eqs. (7) to (10)) are implicit functions of the design variables, given by the analysis equations.

Two types of structures, earth covered and uncovered (Fig. 2) have been considered. The cost function (Eq. (2)) includes: cost of materials (concrete, steel, doors), real estate, subgrade, maintenance, etc. Due to functional requirements the variables {L} are discrete. The behavior constraints restrict the amount of damage due to possible explsions in adjacent structures.

3. The Analysis Model

In the discussion that follows only blast loadings have been considered. It is assumed that other effects (such as fragments) are secondary and may be checked after the optimization process. Modifications in the design can then be made, if necessary. An approximate analysis model has been employed with the following features.

a) Loadings

The explosive materials are represented by an equivalent TNT charge at the center of gravity of the charge [9,10]. The blast loadings due to a possible explosion are computed by the methods described in TM 5-1300 [11]. This model is based on experimental results and idealized (such as piecewise-linear) pressure-time relations.

b) Constitutive equations and resistance-deflection functions

Idealized elaso-plastic behavior models have been employed. Also, it is assumed that cracking (but not spalling) may occur in the concrete. The piecewise linear resistance-deflection functions for the various elements are detemined by the yield line theory, considering dynamic values of material constants [11].

c) Dynamic response of elements

To compute displacements and stresses in the elements the medium is discretized, resulting in systems of lumped masses and nonlinear springs. A single degree of freedom system is assumed for each RC element, with the following equation of motion [11]

$$F - P = K_{Lm}\, ma = m_e a \quad (11)$$

in which F = time dependent external force; P = internal force (resistance); m = mass of the element; a = acceleration; K_{Lm} = a factor relating the value of the actual mass m and the equivalent mass m_e. The value of K_{Lm} is determined from the principal mode of vibration. For example, consider the sector of a plate shown in Fig. 3. The equation of motion (rotation about the support) is

$$Fc - (\Sigma M_N + \Sigma M_P) = \frac{I_m}{\ell} a \quad (12)$$

where c = distance of the resultant force from the support; M_N, M_P = negative and positive internal moments, respectively; I_m = moment of inertia; ℓ = width of the element. From Eqs. (11),(12) we obtain

$$K_{Lm} = \frac{I_m}{c\ell m} \quad (13)$$

For an element consisting of a number of sectors

$$K_{Lm} = \frac{\Sigma(I_m/c\ell)}{\Sigma m} \quad (14)$$

For earth covered elements the corresponding mass of the cover is also considered in the equation of motion [12]. Multi-degree-of-freedom systems may be considered for each element if better approximations are required. Numerical algorithms (such as Runge-Kutta) or available results [11] are used to solve the equations of motion and to evaluate displacements and stresses.

d) Computational considerations

Analysis of a single element, including calculation of the loadings and solution of the equations of motion, involves much computational effort (up to 10 sec CP on IBM 370/168 were reported for complex loading histories). There are several elements in a structure and the analysis usually must be repeated many times during optimization. Therefore, it is proposed to reduce the number of analyses in the solution process by introducing data banks of preoptimized elements for sequences of given loadings and element configurations.

4. Multilevel Optimization

Design of a large complex system usually involves decomposition into a number of smaller subsystems, each with its own goals and constraints [2]. In general, an integrated problem cannot be decomposed into subproblems which can be independently optimized. There may be many different ways of transforming an optimization problem into a multilevel problem. In the model coordination approach [2,7,8] used in this study, we choose certain variables called coordinating or interaction variables to control the lower level systems. For fixed values of the coordinating variables the lower level problems often become independent and simple to optimize. The task in the higher levels is to choose the coordinating variables in such a way that the independent lower level solutions are optimal. The term "model coordination" derives from the circumstance that a constraint is added to the

problem in the form of certain fixed interaction variables.

Define the vector of cross sectional design variables {Q} as

$$\{Q\} = \{TC\},\{AS\} \tag{15}$$

and the vector of configurational and geometric variables {R} by

$$\{R\} = \{D\},\{L\} \tag{16}$$

It has been noted earlier in section 2 that the vector of design variables {X} can be partitioned (Eq. (1))

$$\{X\} = \{R\},\{Q\} \tag{17}$$

In this formulation {R} is the subvector of coordinating variables between the subsystems and {Q} is the vector of subsystem variables, in turn partitioned as follows

$$\{Q\} = \{Q_1\},\ldots,\{Q_i\},\ldots,\{Q_I\} \tag{18}$$

The subvector $\{Q_i\}$ represents the cross sectional variables associated with the i-th element (subsystem) and I is the number of elements. With these definitions, the objective function of Eq. (2) and the constraints of Eqs. (3) to (10) can be expressed in the general form

$$C = F(\{X\}) = \sum_{i=1}^{I} F_i(\{R\},\{Q_i\}) \tag{19}$$

$$g_k(\{X\}) = g_k(\{R\}) \leq 0 \tag{20}$$

$$h_i(\{X\}) = h_i(\{R\},\{Q_i\}) \leq 0 \tag{21}$$

in which g_k are constraints on the {R} variables (i.e., Eqs. (3),(4)) and h_i are constraints associated with the i-th element (Eqs. (5) to (10)). That is, the variables {R} may appear in all expressions, while the variables $\{Q_i\}$ appear only in the constraints associated with the i-th element, and in the corresponding term of the objective function. Specifically, it is assumed that the cross sectional variables of a given element affect only the constraints and objective function component of that element.

The general optimization problem can now be formulated as the following two level problem.

<u>First-level problem</u>. Determine a fixed value for {R} through the constraints

$$\{R\} = \{R^o\} \tag{22}$$

Then the integrated problem can be decomposed into the following I independent first-level subproblems: find $\{Q_i\}$ (i=1,...,I) such that

$$C_i = F_i(\{R^o\},\{Q_i\}) \rightarrow \text{min.} \tag{23}$$

$$h_i(\{R^o\},\{Q_i\}) \leq 0 \tag{24}$$

<u>Second-level problem</u>. The task in the second level problem is to find $\{R^o\}$ such that

$$H(\{R^o\}) = \sum_{i=1}^{I} H_i(\{R^o\}) \rightarrow \text{min.} \tag{25}$$

$$g_k(\{R^o\}) \leq 0 \tag{26}$$

where $H_i(\{R^o\})$ is defined by

$$H_i(\{R^o\}) = \min C_i \tag{27}$$

An additional constraint on $\{R^o\}$ is that the first-level problem has a solution, i.e., that $H(\{R^o\})$ exists.

The two-level problem is solved iteratively as follows:

1. Choose an initial value for the coordinating variables $\{R^o\}$.
2. For a given $\{R^o\}$ solve the I independent first-level problems.
3. Modify the value of $\{R^o\}$ so that $H(\{R^o\})$ is reduced.
4. Repeat steps 2 and 3 until min $H(\{R^o\})$ is achieved.

If all intermediate values for {R},{Q} are feasible, the iteration can be terminated always with a feasible-even though nonoptimal-solution, whatever the number of cycles. This is advantageous from an engineering point of view and may considerably reduce the computational effort, particularly if the object is to achieve a practical optimum rather than the theoretical one.

Since the second level variables ({D} and {L}, see Eq. (16)) are of fundamentally different nature, it is proposed to decompose the second-level problem into two-levels, such that only the configurational variables {L} are optimized in the second level, while the geometric location variables {D} are treated in a new third-level problem. The three problems are solved iteratively until the optimum is achieved. Note that the I first-level subproblems remain unchanged (Eqs. (22) to (24)). The modified second and third-level problems are formulated as follows (Fig. 4).

<u>Second-level problem</u>. For a given geometric location

$$\{D\} = \{D^o\} \tag{28}$$

find $\{L^o\}$ such that

$$C(\{D^o\},\{L^o\},\{Q\}) \rightarrow \min \tag{29}$$

$$g_k(\{D^o\},\{L^o\}) \leq 0 \tag{30}$$

<u>Third-level problem</u>. Find $\{D^o\}$ such that

$$C(\{D^o\},\{L\},\{Q\}) \rightarrow \min \tag{31}$$

$$g_k(\{D^o\},\{L\}) \leq 0 \tag{32}$$

The proposed solution procedure is possible since the system by its very nature can be decomposed. The loadings depend only on the {R} variables, therefore the first level problems can be solved for fixed loadings. The main advantage is that I independent simple subproblems are obtained in this level. It has been noted that $\{D^L\}$ is a function of W, which in turn is a function of {L}. Thus, Eqs. (3) are the only explicit constraints which are functions of both {D} and {L}. The main reason for choosing {L} as the second-level variables is their discrete nature. For any given $\{D^o\}$ only a limited number of {L} values must be considered. The solution process is shown in Fig. 5.

5. Optimization Methods for the Various Levels

<u>First-level</u>. For the relevant ranges of loadings and element dimensions, data banks of preoptimized elements have been prepared. As a first step a sequence of loadings, with fixed peak pressure and variable duration, is computed for a given element configuration. The cross sectional dimensions are optimized by a direct search technique. For each selected concrete thickness the amount of reinforcing steel is minimized, and the optimal thickness is evaluated by quadratic interpolation. The data banks, to be used in the higher level optimization, are obtained by repeating

this process for several element configurations. Intermediate solutions may be evaluated by interpolation.

Second-level. The optimization method in this level is based on a direct search in the space of the discrete variables {L}. For each assumed L_1 value the optimal L_2 is selected. L_1 is then modified, with the current optimal L_2 (or the nearest feasible value) chosen as the initial design. The L_1 values are modified until the optimum is reached.

Third-level. Powell's direct search method [13] is applied in this level with the convergence criterion

$$|C(\{X_q\}) - C(\{X_{q+1}\})|/C(\{X_q\}) < \varepsilon_c \qquad (33)$$

where ε_c is a predetermined parameter and q is the iteration number in the third level. An additional criterion

$$|\{X_q\} - \{X_{q+1}\}| < \varepsilon_x \qquad (34)$$

ensures that the iteration is terminated only if the condition of Eq. (34) holds for two successive iterations.

6. Numerical Examples

Two types of magazine have been considered [14]: earth covered structure with a standard cover thickness and an uncovered structure. Fig. 6 shows some feasible combinations of the {L} variables (a = standard spacing, b = required spacing to satisfy functional requirements, c = required spacing for doors). Fig. 7 shows a typical design space for the cross sectional variables, in which A_s and A_s' are the amounts of reinforcing steel required for tensile and compressive forces, respectively. A direct search in the space of **d** (the effective depth of the cross section), as shown in Fig. 8, provides the optimal element for the given loading and configuration. This procedure is repeated for all relevant loadings and element configurations. Typical optimal costs for a sequence of triangular pulses are shown in Fig. 9, in which p is the peak pressure and the time axis denotes the duration.

Since the second level variables are discrete, the objective function in the third level is not continuous (Fig. 10). It can be noted that local optimum points may exist and the search is terminated only after checking the objective function in a number of points, to ensure that the true optimum is achieved. For each point in the {D} variables space an optimal structure is selected. For larger values of {D} the feasible region in the {L} variables space is increased, and the result might be a change in the discrete optimal {L} values. The computational effort in the second level optimization may be reduced considerably by interactive design. Convergence display programs may be used for this purpose. Fig. 11 demonstrates the relative cost against the number of calculated {R} vectors (Eq. (16)). The latter number is equal to the number of accesses to the data banks. Search in **{D} space is shown in** Fig. 12. Different initial designs or directions in this space provided an identical optimum. Earth covered structures were optimized in a similar manner, with convergence demonstrated in Fig. 13. This type of structure was found to be cheaper than the uncovered structure.

The possibility of choosing the structural configuration {L} as the third-level variables was also checked. While an identical optimum was obtained, the convergence rate was much slower; the number of iterations was about 50% larger. This can be explained by the difficulties encountered in searching the optimum in the discrete variables space. Choosing {L} as the second-level variables, the number of checked points for a given {D} value is limited by the constraints of Eq. (3).

7. Concluding Remarks

A multilevel approach for optimal design of protective structures has been presented. The solution method, which is based on a simplified analysis model and decomposition of the integrated problem into a number of smaller subproblems, is motivated by the following difficulties:

a. Nonlinear dynamic analysis is needed to describe the structural response even when approximate models are used.
b. The various types of design variables are of fundamentally different nature from both the physical and the mathematical points of view.
c. The objective function is neither differentiable nor continuous.
d. The problem size (numbers of variables and constraints) may be large in practical problems.

A simple optimization model is proposed for the cross-sectional variables. Preoptimized elements are introduced for a set of given element configurations and loadings. This information is then used for efficient optimization of the higher level variables. The approach is general and is not restricted to a specific problem, analysis model, or optimization algorithms. Rather, other types of structure (such as steel or composite structures having non-rectangular shape), different analysis models (dependong on the type of approximations and simplifications used) and optimization methods (such as optimality criteria) can be employed.

The numerical examples indicate that efficient solutions which do not involve much computational effort, can be achieved for complex optimal design problems by the proposed approach.

Acknowledgement

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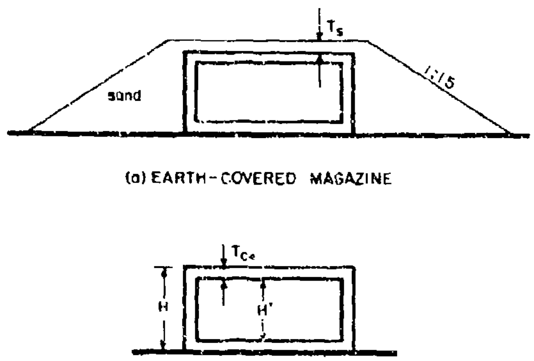


(a) EARTH-COVERED MAGAZINE

(b) UNCOVERED MAGAZINE

FIG. 2

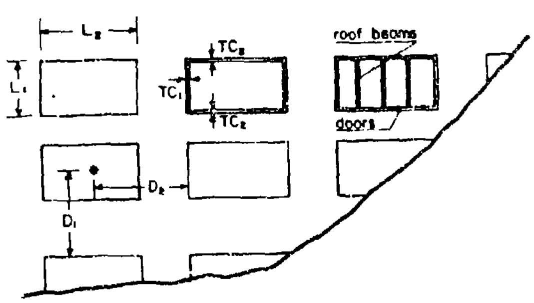
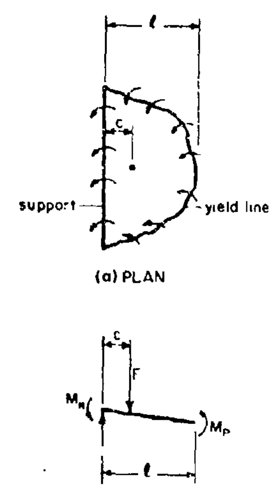


FIG. 1



(a) PLAN

(b) SECTION

FIG. 3

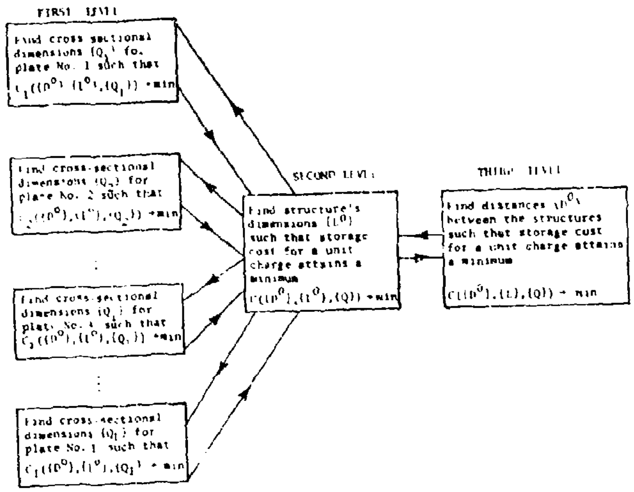


FIG. 4

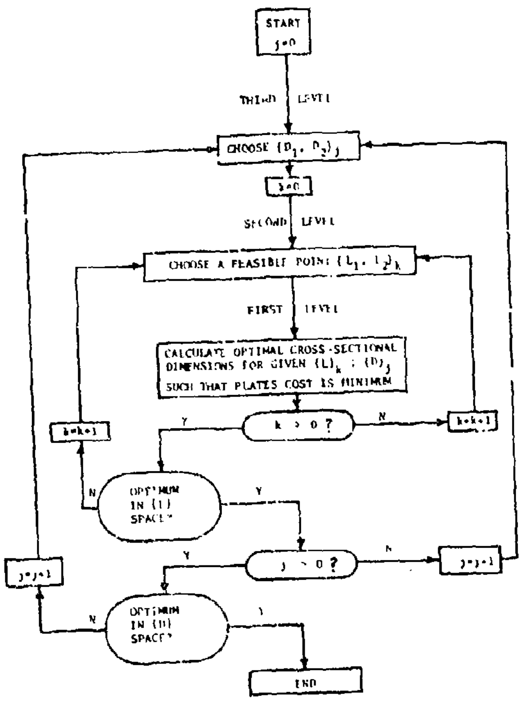


FIG. 5

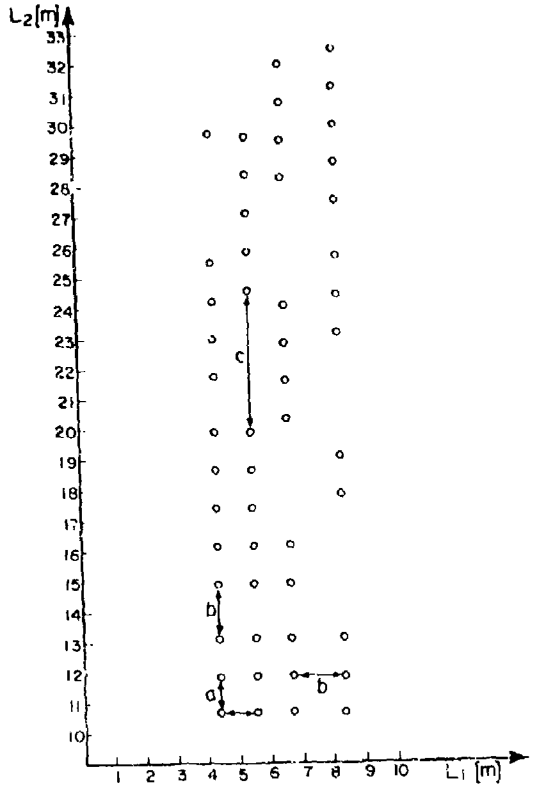


FIG. 6

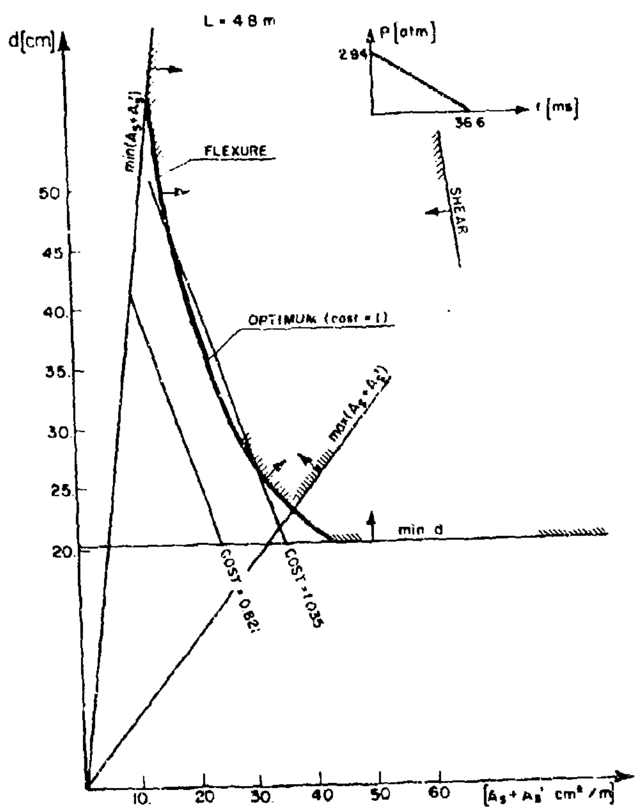


FIG. 7

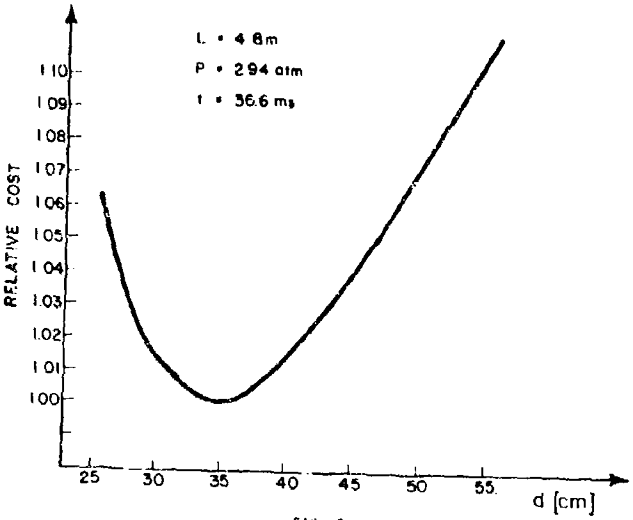


FIG. 8

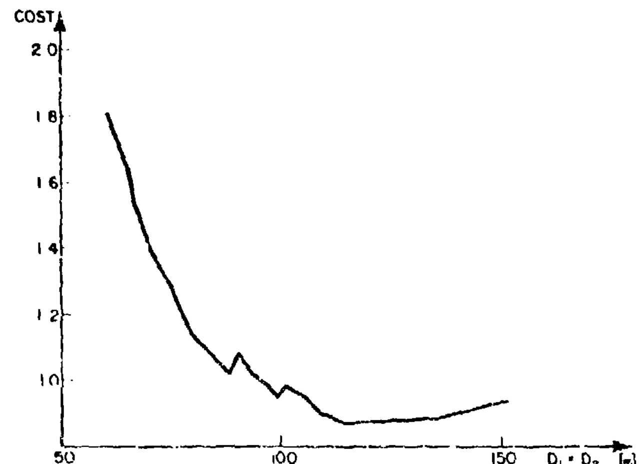


FIG. 10

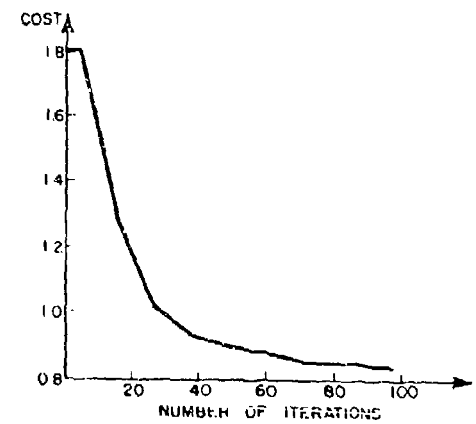


FIG. 11

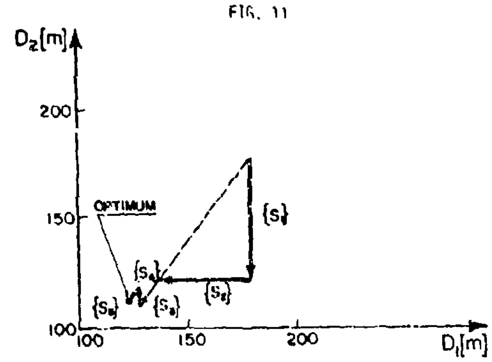


FIG. 12

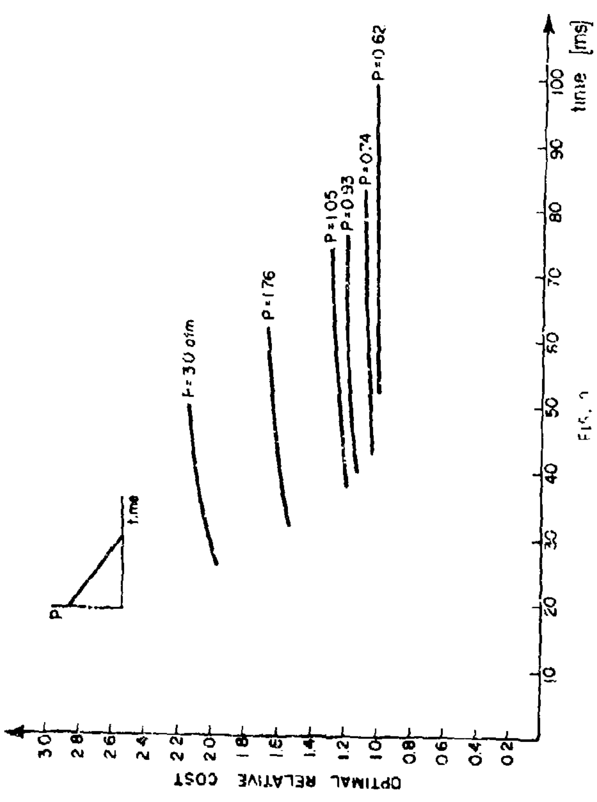


FIG. 9

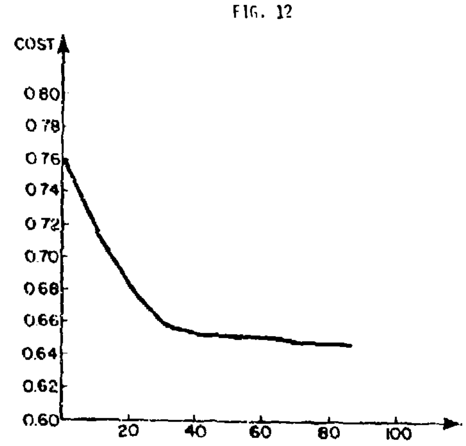


FIG. 13